## Bizarre Arithmetic

ANSWER: THUNDER
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- Look at second column, $0=0$
- Look at the first row of multiplication. In order for it to be the same as the first original number, $\mathrm{T}=1$
- You know that $\mathrm{E}+1=\mathrm{S}$ from the third column
- Since the units digit is always R or 0 , and R is whats multiplied, you know that its equal to either 5 or 6
- You know that $10+\mathrm{C}$ mod10 $=\mathrm{C}$, so from the 7 th column you can see that a 1 or 2 has to be carried over, making H either 9 or 8
- You also know that that $2+\mathrm{S}=\mathrm{H}$ from the 8 th column, and its less than 10 since there is no 9th column
- This makes S either 6 or 7 , and E 5 or 6 .
- But since $R$ is also either 5 or 6 and $E+1=S$, that means $R=5, E=6$ and $S=7$
- That means $\mathrm{H}=9$
- You know that $\mathrm{G}+\mathrm{C}+6+9 \bmod 10=1$, or that $\mathrm{G}+\mathrm{C}=5$
- You know that $\mathrm{C}+\mathrm{A}+6+5 \bmod 10=7$, or that $\mathrm{C}+\mathrm{A}=6$
- $\mathrm{A}+1=\mathrm{G}$
- The only consecutive letters left are 2,3,4
- Use logic and those two equations to find $C=2, B=3, A=4$
- Final word is STONE

| ? |  |  |  |  | M | EZ | $\begin{aligned} & 0 \\ & \mathrm{I} \end{aligned}$ | $\begin{aligned} & \text { W } \\ & \text { N } \end{aligned}$ | S |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |
| ? |  |  |  | S | A | 0 | A | C | C |
|  |  |  | 0 | E | A | N | M | N |  |
|  |  | W | A | A | E | I | I |  |  |
|  | S | M | S | N | N | I |  |  |  |
|  | M | A | C | W | C | N | N | A | C |

- There are 2 ways for $x y \bmod 10=x$
- Either x or y is 5 and the other number is odd. This can only get a units digit of 5 .
- Either x or y is 6 and the other number is even. This can go to any even number.
- Because the units digit is the same for $I, N$ and $C$ when multiplied with $S$, you know that $S=6$ and that $\mathrm{I}, \mathrm{N}$ and C are even numbers. Z is odd.
- At the 8th column, there must be a 1 carried over from the 7th, so $\mathrm{M}=7$
- Because C and N are even and $\mathrm{C}+\mathrm{N}=\mathrm{A}, \mathrm{A}$ is also even
- Because $\mathrm{N}, \mathrm{A}$ and I are even, $\mathrm{A}+\mathrm{M}+\mathrm{I}=\mathrm{N}$ isn't possible without a 1 being carried over. This means $\mathrm{C}+\mathrm{N}>10$
- The biggest number you can get by adding 2 different even numbers is 14 , so A is either 0,2 , or 4.
- $\quad \mathrm{N}$ cannot be 0
- $W$ and $M$ are both odd and $A$ is even, so $W+M=A$ and there is no 1 carried over.
- $0+A+S<9$ since there is guaranteed to be at least a 1 carried over from $\mathrm{S}+\mathrm{E}+\mathrm{A}+\mathrm{N}$
- That becomes $0+A<3$. Since 0 is odd and $A$ is even, the only answer is $0=1$ and $A=0$
- We know then that $\mathrm{C}=8$, since it is even.
- Since $8+N=10, N=2$
- The numbers left are $3,4,5,9$. $\mathrm{E}, \mathrm{Z}$ and W are odd and I is even, so $\mathrm{I}=4$.
- $S+E+A+N \bmod 10=W$ becomes $6+E+0+2=W$, or $8+E=W$. This is only possible if there is an even number carried over from $1+\mathrm{A}+\mathrm{A}+\mathrm{E}+\mathrm{N}=\mathrm{C}$, which is just $\mathrm{E}+2=7$.
- Therefore $\mathrm{E}=5$ and $\mathrm{W}=3$
- Then $\mathrm{Z}=9$
- Final word is OCEAN

6786945. 

|  |  |  | G | L | Y | P | H |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $?$ |  |  |  |  | H | A | Z |
|  |  |  | A | P | I | Y | G |
|  |  |  | E |  |  |  |  |
|  |  |  | H | I | E | L | Z |
| $?$ | Y | E | P | E | W | H |  |
|  | Y | L | H | Y | W | E | L |

- There are 2 ways for $x y \bmod 10=x$
- Either x or y is 5 and the other number is odd. This can only get a units digit of 5 .
- Either x or y is 6 and the other number is even. This can go to any even number.
- $H=6$, and $A$ and $Z$ are even. $Y$ is odd
- $\mathrm{L}+\mathrm{E}=\mathrm{L}$, so $\mathrm{E}=0$. $\mathrm{L}+\mathrm{E}<10$ because it doesn't carry a 1 over.
- In order for $6^{*} Y$ to have a units place of 0 with $Y$ being non-zero, $Y=5$
- $\mathrm{Y}+\mathrm{L}+\mathrm{A}+1=\mathrm{L}$ since $\mathrm{G}+\mathrm{Z}=10$, so $\mathrm{Y}+\mathrm{A}+1=10$ so $\mathrm{A}=4$
- From $1+\mathrm{I}+\mathrm{E}+\mathrm{H}+\mathrm{H}=\mathrm{E}$, or $1+\mathrm{I}+0+6+6 \bmod 10=0$, or $3+\mathrm{I}$ mod10=0, which means $\mathrm{I}=7$
- You know that since G and Z are even, and $\mathrm{G}+\mathrm{Z}=10$, they are 8 and 2 , but you don't know which is which
- You know from $\mathrm{I}+\mathrm{E}+\mathrm{H}+\mathrm{H}+1$, which is now $7+0+6+6+1=20$, that there's a 2 carried over to the next column
- $2+\mathrm{P}+\mathrm{I}+\mathrm{E}+\mathrm{W} \bmod 10=\mathrm{W}$ so $2+\mathrm{P}+\mathrm{I} \bmod 10=0$, so $\mathrm{P}=1$
- $1+\mathrm{P}+\mathrm{L}+\mathrm{P}=\mathrm{H}$, which only works if both P and L are odd. You know that its less than 10 since nothing carries over, so $\mathrm{L}=3$.
- The only odd letter left is W, so W=9
- At this point you should be able to get the word HIGHWAY.

